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A Study on the North Dome of Masjid-i-Jami Isfahan

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Abstract

The North dome, powerful in presence, quietly occupies one of the most secluded areas of the Jami Mosque in Isfahan, Iran. In its silent presence, it narrates a story about an architecture that modestly glorifies both its creators, and in turn, their Creator, This structure is shaped by artists/architects who are not only 'builders', but also 'believers', constructing a building that spiritually serves as a house for God. Although its exact architectural function is unknown to us, its intellectual, artistic and spiritual message is clearly projected in each of its design features. In this paper, the underlying structural integrity and geometric principles in the North dome will be presented. On this basis, the construction of the North dome will be presented in relationship to the mosque complex, the social conditions and also the architectural characteristics of that period. Also, an overview of its unique structural traits and its mathematical relationships will be presented. Lastly, the geometric principles that are employed throughout the construction of the North dome and within its brick structure will be reviewed.

1. Introduction

Throughout centuries Traditional Islamic Architecture has been studied from many different perspectives. In some studies the formation of Islamic architecture has been carried out within the context of history, examining the role that historical events might have played in creating this tradition. By carving into the remnant memories of the (available) past, a set of relationships have thus been drawn between traditional architectural design and its social condition. Almost at the other end of this spectrum, there are studies that have characterized Islamic architecture through their pursuit of symbolic meanings for its different forms. Only rarely has Islamic architecture been examined as 'the act of construction', structural strategy and the role of geometry in the service of both construction and structure.

To understand the construction process that forms an Islamic building, it is important to place it within the context of society and consider the learning and inclinations of people among whom such skills are developed. A case can be made that the traditional Islamic architecture was shaped by craftsmen who were geometers, mathematicians, artisans and philosophers as well as 'builders' of these structures. A traditional artist *(Muhandis)* uses the principles of geometry to construct his art. Whether it is to weave a carpet or to construct a building, the arrangements have come together by the very same principles and yet are unique in form. From a carved wooden door in a simple house to an elaborated ceiling in a Sultan's palace, from an elegant structure of a Grand Mosque to the intricate fabric of a city, the construction strategies might individually vary, but collectively, they carry the same underlying ideas and geometric principles.

In this regard, to study any form of Islamic architecture we are confronted with questions such as how these constructions are built, strategized and finally realized. How are geometric principles applied?

What role does geometry play in transforming an idea into a constructed form? What materials and techniques were applied to construct these forms?

To cope with such questions a systematic approach is needed. One such approach is presented in this paper through exploration of the underlying structural concepts, geometric principles and construction techniques in one of the most celebrated examples of traditional Islamic architecture: the North dome of Masjid-i-Jami of Isfahan, Iran.[1]

2. The General Layout

The Jami mosque is the first and thus the oldest congregational mosque for the city of Isfahan in Iran. This remarkable structure has survived for over one thousand years and is still being used with its original purpose. As Oleg Grabar remarks, a work of architecture with such quality has a contemporary meaning that reflects on some of our contemporary ideologies.[2] In this regard, the value of a building that is permanently alive and functional transcends the frozen concepts of a historical monument and becomes a living phenomenon. Historically, Masjid-i-Jami was mainly constructed during the period known as the Saljuq dynasty in Iran. The Saljuqs came to power around 11th century when the Persian civilization after four centuries of Islamic rule had become intellectually quite advanced.[3] It was during this era of learning and knowledge when various libraries, '*kitab-khaneh*', literally meaning 'abodes of books' were established everywhere. These libraries were open to the public and therefore, learning was a common value shared by citizens. From this there emerged a series of scholars, scientists, designers and artists who offered considerable achievements in many fields of science and arts.[4]

In the general layout of the Jami Mosque, there exist two domed chambers along a 'north-south' axis i.e. the Qibla orientation. One is located between the South iwan and the *mihrab*, and is known as *Gunbad-i-Nizam al-Mulk*. The construction of the South dome was done in about 1070-1071 A.D. The south dome is a square room of about 16 meters (50 ft.) and nearly 30 meters (90 ft.) high. This spacious place supports a huge dome of about 50 feet.

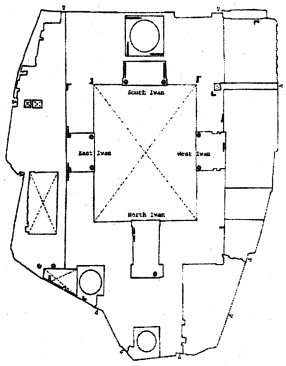

Figure 1: *Masjid-I-Jami Plan* [5]

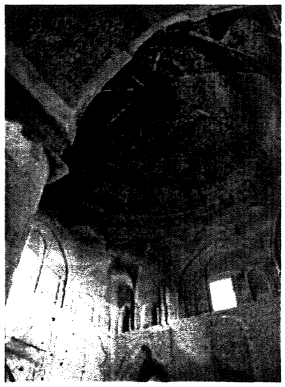

Figure 2: *South Dome Interior View*

In the opposite direction from the South dome, there exists another dome structure, smaller, yet more elegant than the south dome. This dome, which is the primary subject of this study, is located on the north side and is known as the North Dome, also known as *Gunbad-i-Kharka*[6], or *Gunbad-i-Khaki* (the earthly dome),[7] and Gunbad-*i-Taj-ul-mulk*.[8] Aesthetically, it is a magnificent work of art and is one of the most important units in the Jami Mosque. This structure is the only part of the mosque which has the exact date of its construction written on the rim of its dome-dated 1088 A.D. The construction of the North Dome started one year after the South Dome.[9]. In the forthcoming discussion, though a systematic framework, the architectural characteristics of the North Dome will be examined on the basis

of its historical context, structural complexity, constructive elements and finally its geometrical formation.

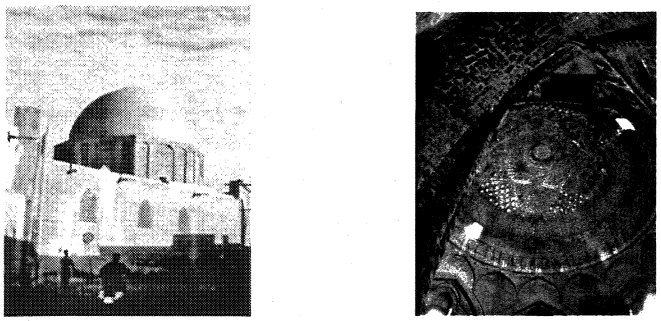

Figure 3: *Exterior view of North dome (Left), Interior view of North Dome (Right)*

The construction of the North dome creates a paradox for anyone studying the Jami Mosque. Traditionally, it is not common to have two distinct domed structures in one unified mosque. In a layout such as the Jami mosque where a grand dome exists aligned with Qibla and is closer to the Mihrab, (Refer to Fig.1) it seems unnecessary to have another dome further back on the opposite side of the axis.[10] Why was the North Dome constructed and what was its original function? There exists no specific answer to these questions, however, throughout the history a few possibilities are speculated that suggest the function of this structure. Some believe that this dome was constructed as a library for the mosque. Although, the dome's location close to a major public entrance makes this theory a weak assumption. Some say it was a quiet place of meditation for either the King or the Queen. The problem with this theory is that normally, the seat of the rulers are respectfully placed close to the Qibla wall, which is located at the south end of the Jami Mosque, but the North dome is on the opposite side. Also there is a possibility that it was used as an astronomical observation site by some great scholars. Despite all these speculations, there is no definite answer to our question.

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Architecturally, the foremost characteristic of the Saljuq buildings was a very refined sense of masonry, especially in brick structures and patterns. Although many examples of brick construction existed before, Saljuqs carried this material and its techniques to near perfection. They mastered techniques to employ brick as the main material for construction as well as for ornamentation. The uncovered brick was applied to display pure structural elements, while it was also artistically combined with ornaments. The balanced coordination of structure and ornament in their brickwork lent a uniquely honest and characteristically expressive personality to their architecture. Saljuq brickwork can be described as a combination of inspiration and craftsmanship. The surviving structures of this period are a testament to the craftsmen and builders, whom with their extensive knowledge made countless geometric forms in brick. Some of these executed patterns were combined with richly inventive carved stucco and terracotta. Others were elaborated forms shaping structural elements.

Another Iranian invention, which flourished during the Saljuq period was a technique to transform a square chamber into an octagon on which a dome could be placed. This resolution, commonly known as squinch, is called *Goshwareh* in Persian, which literarily means 'overturned or transformed corner'. It was a method used prior to the Saljuqs in Iran and Central Asia, but Saljuqs perfected this construction technique and raised it from a mere solution to a problem of geometric transition to an artistic device of high order. This invention soon became the basis for one of the most significant structural elements named *Muqarnas*. In later periods, the Muqarnas vault system evolved to not only enhance structural support for squinches but also to impart a characteristic expression to them. Structurally, the process of making a squinch involves mechanical techniques, geometrical transformations and mathematical

analysis. Saljuqs, by their very careful observation, were able to bring these three elements together to achieve what could genuinely be called an "architectural alchemy".

Builders of this period also made major contributions to another structural problem. Domed structures have been a unique characteristic of Persian architecture even in pre-Islamic periods.[11] Saljuqs, however, managed to acquire mechanical efficiency in their dome usage that was hitherto unknown. Saljuq builders developed techniques that reduced the mass of their dome, while achieving geometrically defined outlines and pleasing proportions. By using mathematical and geometrical principles these builders were able to achieve their goal.

3. Structural Significance of the North Dome

Structurally, in many architecture books the North dome is introduced as one of the best masonry domed structures ever built. The proportions of this dome are based on the Golden ratio [(1+ /5)/2] and the dome's characteristic profile has become synonymous with Saljuq form. This building combines the pleasing mathematical / geometrical proportions with the lightest structural form, and structurally suggests a good example for a perfect domed construction. [12]

Over 900 years of history has proven that the structure of the North dome is the only segment of the Jami mosque that has survived earthquakes, fires and civil wars with no considerable damage.

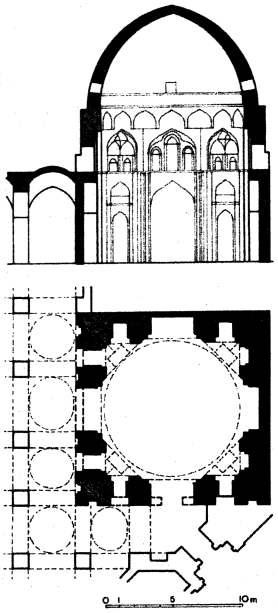

Figure 4: *Plan and Section of North Dome* [13]

While it may be generally assumed that 'structure limits form', if one looks at Masjid-i-Jami's North dome, structure opens up new possibilities of expression. "Beauty of form and mechanical perfection of structure are by no means identical terms"[14]. But in this building these two elements are well combined in a perfectly balanced form and therefore, the unity is silently achieved.

The North dome has two very distinct qualities when it is approached from the outside, or experienced from the inside. Like many other forms in Islamic architecture, the outside volume is relatively plain and silent, whereas, the inside space is extensively detailed and expressive. The North Dome structurally stands independently from the rest of the mosque complex. Visually, the North Dome is surrounded by a series of arcades in the south and the east walls. The basic structure of the North Dome is based on a

form known as *chahar-taq*[15] in Persian architecture. This form consists of a domed roof on top of a square base with four load bearing columns in each corner. This unique structure was originally developed in pre-Islamic Zoroastrian fire-houses and in later periods, was adopted as the basic form in Islamic tombs.

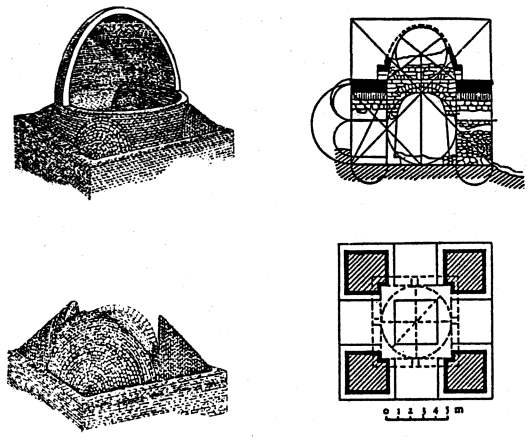

Figure 5: *Chahar-Taq Structure* [16]

In the floor plan, the North Dome can be divided accurately into three equal sections along the edges of the square base. By this division a nine square grid will be formed which in itself has an important significance in Islamic geometry and mathematics.[17] The division of nine square units is such that the middle arches on each side are aligned with the middle section, and the two smaller arches are aligned with the corner sections. Along a vertical section, Gunbad-i-Kharka can be divided into three major parts. The first part is a square chamber, located in the lower section of this structure. This volume is completely enclosed from the north and the west walls and has openings in the south and the east walls. The interior space of this chamber is signified by a series of arches that visually lift the structure upwards and give a lighter feeling to the interior space. Each side of the square chamber is characterized by three major arches, wherein the middle arch has a wider span than the other two and visually dominates the space.

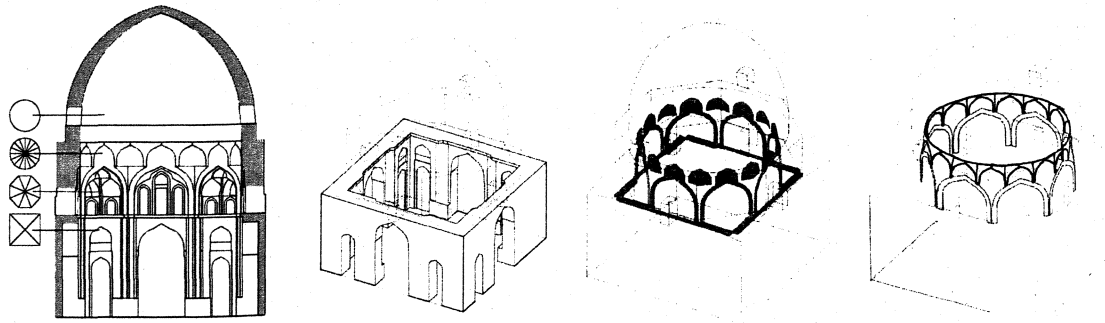

Figure 6: *Geometric Divisions at the base and Zone of Transition*

The second (middle) part of the structure is a section that stands between the lower square chamber and the dome itself. This section is the result of a series of geometrical transformations that start from a square base, then form an octagon (eight sided polygon) and by the time they meet the circular base of the dome, form a hexadecagon (sixteen sided polygon). This middle section is considered "the zone of transition" which in itself is an innovative design. In the zone of transition, the octagonal volume is formed by eight large arches of equal profile in each plane. Four of these arches are aligned with the planes of the square chamber (below). The other four are located at 135 degree angles, at each of the

corners, completing the octagon. The corner arches frame four squinch structures. These squinches are the basic structures that transform the square base to the circular dome. The north dome squinch, when viewed from the front, has a trefoiled or three-lobed arch. The profile of the side lobes revolves around its peak with a linear base. The middle lobe, on the other hand, revolves 180 degrees around its peak, with two perpendicular lines forming its base. The result of this arrangement is a unique structure that brings surfaces and revolved forms together in a geometrical space. Between these arches an element emerges to allow the smooth transition from the octagon up to the round dome. Above the great arches of the octagon this element expands like a flower and comes to an end at an altitude where the octagon ends and a hexadecagon starts. At this point, a ring of sixteen arches is repeated around which the circular rim of the dome is placed.

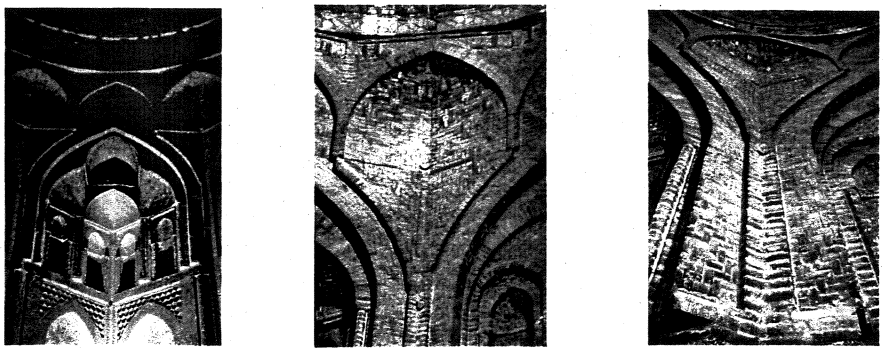

Figure 7: *Squinch Structure*

The third part of the Gunbad-i-Kharka's structure is the dome itself. The rim of the dome is situated on top of a cylindrical drum, which is decorated with a complete ring of calligraphy. The interior space of this domed structure is fully designed with geometrical patterns that are based on pentagons and triangles. The domed structure adds two more windows to the interior space that are located along the north-south axis.

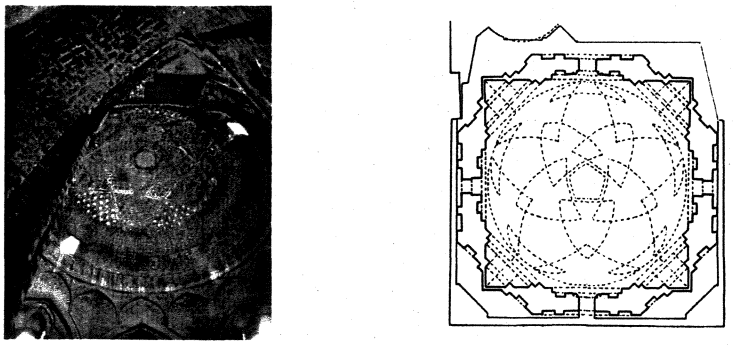

Figure 8: *Dome Structural Pattern* [18]

4. Geometrical Significance of the North Dome

As mentioned, by the 11th century, the Islamic cultural life and thought had reached the point where it was highly advanced both aesthetically and intellectually. This advancement was truly the product of the new faith (Islamic religion), whereby acquiring knowledge was ranked second after prayer and therefore, learning was regarded as one of the most sacred acts for a pious Muslim. In this respect, "nature was to be studied in her mode of operation and not merely in her manifested form" [19] and various sciences and arts were to be admired as a means for better understanding of the universe which includes both the world that is known to us *(alam al-shahadah)* and the one that is invisible *(alam al-ghayb)*.

Likewise, this unique approach influenced the attributes of architecture as much as it affected other fields of science and art. In this regard, Islamic architecture became an abode to embody the sacred art, and Islamic geometry became its instrument to give it a physical form. For Muslims who were impressed by the Pythagorean concept of space on the one hand and Divine creation on the other hand, the Islamic concept of geometry was an instrument to establish a sense of sacredness in art and architecture.[20] On this basis, the structure of the North dome was formed using geometric order(s) in different levels of construction.

In the first level, geometry is incorporated to form the basic structure. As explained earlier, the plan starts as a perfect square and along a vertical axis, it gradually transforms into an octagon (eight sided polygon), and then into a hexadecagon (sixteen sided polygon) until it meets the circular rim of the dome. Throughout this transformation, corner squinches have developed which have their unique geometric form. The geometrical structure of the North dome, although externally solid and plain, internally creates a unique geometric space. This space visually provides other geometric attributes such as symmetry, structural balance and harmony.

In the second level, the geometric order of the North dome is based on a precise mathematical proportion. The square base is divisible into a close pack of nine square units, such that each of the arches of the square chamber are proportionally aligned with this division. Similarly, in section, the main structure is carefully proportioned based on the mathematical laws of the Golden ratio.[21] This division has been further carried out to another level, where forms such as an equilateral triangle, a lozenge, and a pentagon create a greater geometric relationship between different sections of the structure. This deliberate use of mathematical relations is an attempt to create a dialogue between natural forms and man-made architecture. To a Muslim believer, the creation of the world is based on the multiplication of natural phenomenon. God, who is 'One', is the beginning of everything and therefore, under His command creation starts from 'two' and is carried to infinite numbers. The generation of numbers in mathematics is similar to the basic rules of creation. In this respect, numbers are closely bound to nature. Using this analogy, since form is quantifiable through mathematics, then, geometric forms are related to nature via the abstract language of mathematics. Based on this indirect relationship, the use of geometry as "the expression of personality of numbers" permits further exploration into the process of nature.[22]

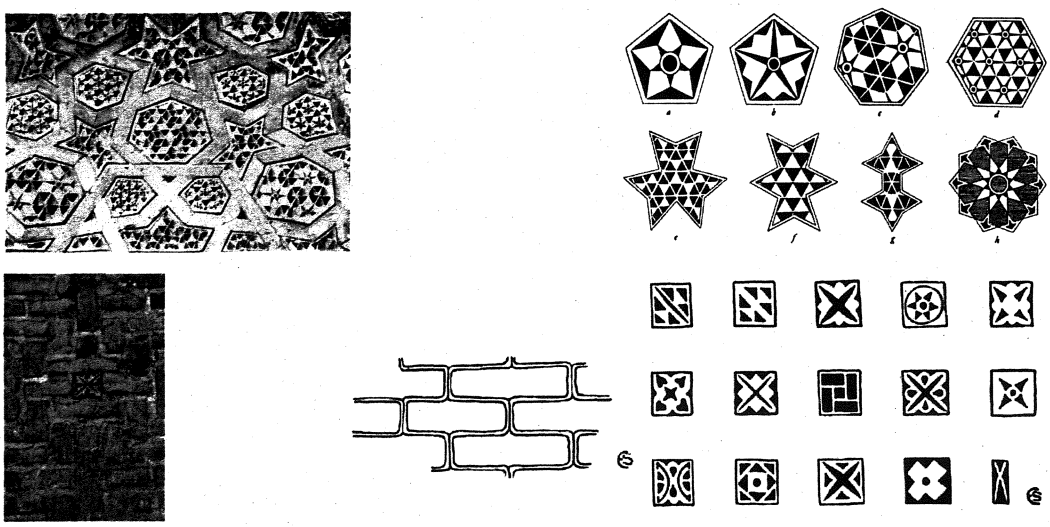

Figure 9: *Some of the geometric carved out patterns on the surface* [23]

At the third level, the geometric forms reveal a symbolic meaning beyond their physical application. On this basis, geometry bridges the gap between conception and perception, such that it transforms an

intangible idea into a tangible form. Much literature has been written in respect to this geometric trait. One work in particular that is presented here corresponds closely to the structure of the North dome:

> *"Various geometric forms have specific symbolic meaning which relates the outward forms to inner meaning and architectural utility to spiritual significance. The dome, while creating a ceiling which protects from both heat and cold, is also the symbol of the heavenly vault and its centre the axis mundi which relates all levels of cosmic existence to the One. The octagonal base [refer to the zone of transition] symbolizes the Throne and Pedestal and also the angelic world. The square base [corresponds] to the corporeal world on the earth. The vault structure [of squinch] represents reflection here below the supernatural archetypes, the descent of the heavenly abode towards the earth and the crystallization of the celestial substance or ether in terrestrial forms... [theses geometric] forms create a sacred quality and signify realities beyond the earthly realm."*[24]

Lastly, geometry serves a decorative function, as it covers the entire surface of the North dome with aesthetically pleasing patterns. Historically, the art of decoration on the surface of Islamic buildings started in the early periods of the Islamic architecture. For a Muslim artist, ornamentation was not a mere representation of living forms; instead, it was a symbolic language with cosmic references. This language was expressed by means of geometric patterns, arabesque forms and calligraphic panels filling the surfaces of walls, ceilings and sometimes even floors.

5. Geometric Patterns

The main geometric patterns were created by bricks that are molded in various forms. These bricks have distinct geometrical arrangements in different sections of the building. Some of the brickwork is purely decorative, while other brickwork participates in the structural stability of the building. Both structural and non-structural bricks are harmoniously combined in such a way that they cannot be easily identified from each other. The final decorative design to the interior surface is given by carved terracotta ornaments and plaster inlays. Both of these materials are capable of producing small decorative forms that are visually organic to the brickwork and easily combine with it.

Geometric patterns on the interior surface of the North dome are developed from the repetition of a geometric form (which can be a polygon or a combination of different polygons) in and around a circle.[25] In this respect, the circumference of a circle can be divided into 3,4,5,6,... equal sections and various geometric forms can generate from this circle based on such divisions.

In addition to the surface patterns, brickwork in the North dome also forms a series of calligraphic panels. Historically, calligraphy has been vastly used on the interior and exterior facades of Islamic buildings, particularly for the religious architecture such as mosques and tombs. Most of the calligraphy writings are the selected verses from the Quran and they are used to highlight particular concepts. Calligraphy in this sense is a direct expression of the Divine; by means of its carefully orchestrated use the building virtually becomes "the Word of God".[26]

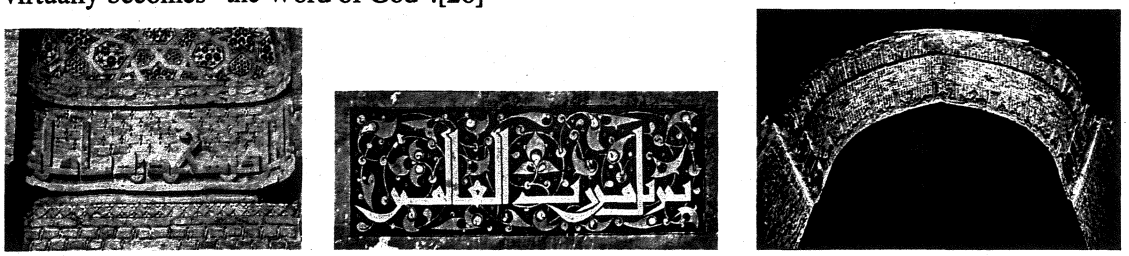

Figure 10: *Some example of calligraphy work in the North Dome*

Besides its spiritual message, the Arabic calligraphy by its very nature is also well adapted to decorative developments. Due to its unique writing principles, it is compatible with the geometric patterns. Arabic

letters are formed based on the combination of verticals and horizontals in a continuous design and so lend themselves to abstract decoration. "The verticals provide structure and rhythm, the horizontals impart balance and continuity. [Moreover], the necessity of joining certain letters promotes the tendency towards the creation of complex traceries".[27]

The very structure of Arabic calligraphy also provides a cosmological symbolism based on the composition of horizontal and vertical strokes. "The verticals, like the warp of a carpet, provide an ontological relationship as well as structure for design, while the horizontals, like the weft, correspond to the creation that develops the balance and flow of the basic conception. It is through the harmonious weaving of the horizontal and the vertical that unity is achieved."[28]

The styles of Arabic calligraphy have been modified throughout time; however, the basic principles have remained practically unchanged. The calligraphic style that is used for the North dome is known as Kufic. This style is one of the first Arabic scripts that was developed by Muslims during the early periods of Islam. Compared to other styles, Kufic calligraphy has a unique geometry and proportions and therefore, can be composed in various scales and be adjusted with geometric patterns.

6. Brick Construction of the North Dome

The construction of the North dome is directly related to the building methods that were used by Saljuq builders. Thus, to understand the techniques applied to this dome, it is necessary to look into the construction methods from this period. It is important to state that the information about the traditional construction of Islamic architecture in general and that of the Saljuqs in particular has not been documented well and is not available. There exist few studies that go beyond illustrating geometrical rules to actually describing construction methods.

As mentioned, brick was an accepted building material for most of the Saljuq architecture. Burnt clay brick originated in Persia in the third millennium B.C.[29] When Persian craftsmen first used brick, new possibilities of construction were discovered and soon the builders had good reasons to prefer brick over other materials. The advantages of brick were obvious and substantial. In comparison with stone, brick was less expensive and in comparison with wood more enduring. More importantly, considering that arid and hot climate covers major parts of the region under the study, [30] brick is most abundant. Brick also allows rapid construction. Because of its superior "elasticity", it is capable of forming in different ways. "This elasticity is a necessary consequence of the smallness of the units of composition which permits adjustments individually minute and invisible, but cumulatively demonstrative and emphatic."[31] Structurally, each brick unit demonstrates physical forces and aesthetically, it can be molded in different forms. Brick with its unique qualities opened a field of exploration for Persian builders for many centuries.

Although brick had been used for centuries before the Saljuqs, with Saljuq architects brickwork was carried to unprecedented perfection both aesthetically and constructionally. The builders of the Saljuq era explored the possibilities of applying brick in a "pure style" in such a way as to exclude all other major materials of construction. Brick was used both as the structure for buildings as well as ornamentations. By this application, they explored fully all aspects of this material. The North dome of Masjid-i-Jami is a prime example of exposed brickwork and brick construction during the Saljuq era. "Here the beauty of the exposed form is supreme and the ornament one with the substance of the structure and inherent in the material"[32]

The construction of the North dome is an interweaving of various brick forms in a harmonious setting. From the base to the dome's top, from the inner surface to the exposed form outside, layers of brick are put next to one another in different directions to establish this form. Although visually the brickwork is

unified, it structurally plays different roles in each section. In some parts where the structural load is concentrated, the brick of higher q[33]uality is used in higher density. Where the construction is lighter, the brick is applied with more elaboration and detail. Also, the brickwork on the dome's inner surface is smaller and uses more design varieties, whereas on the exterior surface the brickwork is laid using a much simpler design.

7. Conclusion

The North dome of Masjid-i-Jami of Isfahan, powerful in presence, quietly occupies one of the most secluded areas of the mosque. In its silent presence, it narrates a story about an architecture that modestly glorifies both its creators, and in turn, their Creator. This architecture is shaped by artists/architects who are not only "builders", but also "believers", constructing a building that spiritually serves as a house for God. Although its exact architectural function is unknown to us (other than the fact that it is part of the Jami Mosque), its intellectual, artistic and spiritual message is clearly readable in each of its design features.

In its structural design, by employing mathematical relations that are driven from an idealized nature, i.e. the Golden ratio, this elegant form creates a connection between natural and man-made structures. Also, based on these relations, a 'perfect' form is constructed that not only structurally speaks of harmony and balance, but also speaks of formal perfection and timelessness, which is comparable to the ever lasting faith in a believer's heart. This structure has survived for more than nine hundred years in an earthquake zone and is unchanged from its original form.

In its geometric design, the North dome introduces a unique relationship between structural form and ornamentation. In one instance, geometry is used to create a two-dimensional surface pattern. In another instance, by elevating a surface into a three-dimensional structure, creates a space. When entering the space of the North dome, one experiences the main structural geometry. Gradually, countless geometric patterns appear. Looking further, one discovers more detailed forms that were otherwise hidden. The geometric design is enhanced as the structure rises through successively elevating zones. This quality invites the observer to raise eyes upwards to the point where the dome is placed. The star-like design of the dome is significantly comparable to the heavenly sky; by means of which, one can be connected to the spiritual realm. In this sense, geometry becomes an abstract cosmological language that is revealed in both structural and aesthetic forms to create a rapport between architectural form and the universe.

In its construction, each of the brick units is an individual character that collectively forms the total design. Each brick unit is uniquely placed in its assigned location. Some bricks are purely structural and have a simple form. Others are purely decorative and are fashioned with elaborated geometric designs. Similarly, there are bricks that plainly cover the floor, while others with intricate design cover the curved surface of the dome. Finally, brickwork is also used for the Kufic calligraphy on various panels, where it becomes a calligraphic stroke to celebrate God's Word. A great work of art like the North dome is a lesson for us to see how architecture tells a story and conveys a message by incorporating different design components, from structure to geometry and construction. By these means it collectively brings a form into life that geometrically speaks of harmony, structurally speaks of knowledge, aesthetically speaks of art and above all, silently speaks of faith and subtly glorifies the Unseen.

Lastly, the space inside the North dome is a cut out from the material world around and is filled with a comforting silence that can only be captured through "the loud flight of agitated pigeons leaving a profound silence [behind]". This space silently speaks of many qualities that simply cannot be captured in a work like the one that was presented here. However, it is hoped that this humble effort will open new possibilities for further explorations, in pursuit of many issues that have developed based on this study.

[1] The word *masjid* literary means mosque and *Masjid-i-Jami* is a term used for the major mosque in any Islamic city. *Masjid-i-Jami* of Isfahan is also known as *Masjid-i-Jom'a* which means Friday Mosque. There are other names also associated with this mosque such as *Masjid-i-Jami-i-Atigh* which literary means the Old Congregation Mosque. This name was possibly given in later years, after the city was developed and other Mosques were built.

[2] Oleg Grabar, The Great Mosque of Isfahan, London, 1990, pp. 13.

[3] Islamic religion was established around 600 A.D.

[4] Among all the scholars of this period we can name three who are known worldwide. First, *Nizam al-Mulk,* the wise minister of Saljuq's Sultan *(Malik-Shah)* whose name is written on one of the calligraphic panels in the south dome of Jami Mosque. Second, *Al-Gazali* (1038-1111), the great theologian, philosopher and the man of great power whose books are known and studied by other great scholars; and lastly, *Omar Khayyam*, the great Persian poet, philosopher and mathematician.

[5] left image original drawing from Eugenio Galdieri, "Esfahan: Masgid-I Gum'a", Persian Translation.

[6] *Gunbad* means "dome" in Persian and *Kharka* is the name that is given to this dome.

[7] Kharka+Kharga means a robe that is worn by Darvish and Gunbad-i-Kharka is one of the names given to the North dome. Another name that locally is given to this dome is *Gunbad-i-khaghi* , meaning an egg form dome. *Khaghi =Khaghineh* (a dish that is made with egg).

[8] The North dome was built on the order of Taj-ul-mulk, one of the ministers of Malek Shah. His full name in Persian is inscribed on a frieze of calligraphy around the rim of the dome.

[9] If the construction of the North dome had started one year after the South dome, then, the building had been under construction for about seventeen years. (1071-1088 A.D.)

[10] In most of the mosques of traditional Islamic architecture, the opposite side of Mihrab (Qibla wall) is marked by a small tower, or in some cases a Minaret. Such examples are as follows: The Mosque of Baibars in Cairo; the Great Mosque of Samara; and Qarawiyin in Fez.

[11] Historically, domes were used in Persian Fire temples before Islam and they were extensively used in shrines, tombs and mosques architecture after Islam.

[12] E.Schroeder in his chapter on Saljuq architecture in "*A survey of Persian Art*" describes the quality of the north dome as follows "Quite early in this period (referring to the Saljuq period) there emerges in Central Persia a dome which approximates with almost inexplicable closeness to the "ideal" dome which could only be prescribed after the advance of mechanical science under Newton. It was a dome of steeper pitch than any that the Roman or the Eastern world had hitherto seen".

[13] Drawing from John d. Hoag, *"Islamic Architecture"*, page 194.

[14] A.Pope, "Persian Architecture".

[15] The word *"chahar-taq"* is a Persian word. *"Chahar "* means four and *"Taq"* means cover or curved ceiling. Another meaning for *chahar-taq* is a room that is built on top of four columns, *Burhan-i-ghatiq.*

[16] Images from, G. Memarian, :The construction of arched forms in Islamic Architecture of Iran", Iran press.

[17] The magic Square is a well known mathematical problem that is based on nine square grid. Also, in Islamic geometry the close packing of nine square units is used as the base for many geometrical patterns. Similarly, this form has been used for the basic layout in some of the forms of Islamic architecture. References: J. Berggren, "Episodes in the Mathematics of Medieval Islam". N. Ardalan, "Sense of Unity", G. Haider, "Islamic Architecture and Cities".

[18] Right side drawing, originally is from Eugenio Galdieri, "Esfahan: Masgid-I Gum'a", Persian Translation.
[19] Ardalan & Bakhtiar, *"The Sense of Unity"*, The Sufi Tradition in Persian Architecture.

[20] Geometry by its very nature resides between the world of science and art. Driven as a branch of mathematics, geometry is based on theoretical science and deals with the properties and relations of lines, angles, surfaces and

solids. Geometry also belongs to the world of art and imagination where theses properties can be formed in endless arrangements.

[21] A thorough study by Eric Schroeder is done on the mathematical proportions of the North Dome.

[22] In the Islamic philosophy, a branch of science known as the science of numerology is developed which deals with the numerical symbolism of letter, *"ilm al jafr"*. On this basis, the creation of world, or the manifestation of existence is compared to the generation of numbers and therefore, numbers are closely bound to nature. Geometry is driven from mathematics. Number 1 symbolizes a point, 2 symbolizes a line, 3 symbolizes a surface, and 4 symbolizes the first solid form which is the first "platonic solid" known as Tetrahedron.

[23] Drawings on the right side are from, E. Schroeder, A survey of Persian Art".

[24] Nasr S. H. , "Islamic art and Spirituality". Pages 49-50

[25] The main regular polygons that are recognizable on the surface of the North dome are triangles, squares, pentagons, hexagons, octagons and dodecagons. Only three of theses geometric forms (triangle, square and hexagon) can fill a surface with close packing of themselves, since multiples of their vertices add up to 360 degrees. Nevertheless , different polygons can combine with each other and generate a system of closed patterns with infinite numbers and arrangements.

[26] E.Dood & S.Khairallah, "The Image of the Word", Vol. I and II, 1981

[27] A.U. Pope, "Calligraphy; An Outline History, A survey of Persian Art".

[28] ____ ibid ____

[29] Arthur Upham Pope, "Persian Architecture" and Hossain Zomorshidi, "Traditional Construction Material in Persian Architecture"

[30] The central part of Iran to the east is all dry and has an arid climate (Isfahan geographically is located in central Iran). Towards the south, the weather progresses to a more humid and hot climate. Along the Caspian Sea towards the north, is the only area with moderate climate and therefore, there exists numerous trees and plants in this region. As a result of an expanded hot and arid climate, most of the land is covered with different types of earth and sand. In this respect, the essential material for making brick is easily available to the builders of this region.

[31] Schroeder, "a Survey of Persian Art", the architecture of Saljuq period, A.Pope, 1939.

[32] _____ ibid _____